EOTA

TECHNICAL REPORT

Worked example calculation of characteristic load-carrying capacities of 90° angle bracket with a rib

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Foreword

EOTA Technical Reports are developed as supporting reference documents to European Technical Approval Guidelines and can also be applicable to a Common Understanding of Assessment Procedures, an EOTA Comprehension Document or a European Technical Approval, as far as reference is made therein.

EOTA Technical Reports go into detail in some aspects and express the common understanding of existing knowledge and experience of the EOTA bodies at a particular point in time.

Where knowledge and experience is developing, especially through approval work, such reports can be amended and supplemented.

When this happens, the effect of the changes upon the European Technical Approval Guidelines will be laid down in the relevant comprehension documents, unless the European Technical Approval Guideline is revised.

This EOTA Technical Report has been prepared by the EOTA Working Group 06.03/01 – "Three dimensional nailing plates" and endorsed by EOTA.

1 Scope

To illustrate the guidelines for static models and calculations given in the ETAG 015 "Three-dimensional Nailing Plates", an example of a 90° angle bracket with a rib, where the calculations are assisted by tests, is worked through.

2 Introduction

The ETAG 015 details three methods of determining the load-carrying capacity of a three-dimensional nailing plate connection in a timber structure by

1 – calculations

2 – testing

3 – calculations assisted by testing.

The first method can be used where a common method of calculation has been established.

The second method has been used for a long period but is elaborate and costly.

The third method is based on a calculation method calibrated or documented by tests.

Advantages include the ability to document the same high load-carrying capacity as can be found by testing, and cases other than those tested can be calculated, eg other nailing patterns or loading eccentricities can be covered.

Since this method is relatively new in Europe, by way of demonstration a simple example is given here of a 90° angle bracket with a rib subjected to a lifting force or to a shear force in the direction of the purlin.

In the absence of ETA's and agreed CEN Standards, references have been made to national standards and approvals.

These will have to be substituted by European technical specifications in the final calculations.

Static calculation methods have been proposed and documented in EOTA Technical Report TR 015.

Based on these calculation methods, the characteristic load-carrying capacities of the bracket subject to a lifting force F_1 and a shear force in the direction of the purlin F_2 have been calculated according to the principles of Eurocode 5.

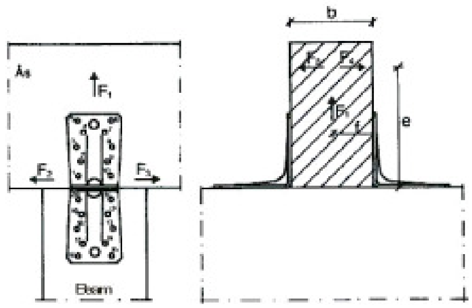


Figure 1 – 90° angle bracket with a rib - nailing pattern; direction of forces relative to the supporting beam and the purlin

The characteristic load-carrying capacities F_k are intended to be used for the calculation of the design capacities F_d in the following way

$$F_d = \frac{F_k K_{mod}}{\gamma_M}$$

where

K_{mod} = factor considering load duration and climatic conditions

γ_M = partial safety factor for the material.

3 Basis of design

3.1 Forces and geometry

Two angle brackets are used at the connection.

The geometry of the angle bracket is shown in Figure 2. The directions of the forces are shown in Figure 1.

The characteristic load-carrying capacities are determined for a lifting force F_1 and for a force parallel to the purlin F_2.

F_1 is situated in the middle of the purlin.

F_2 is situated at the bottom of the purlin.

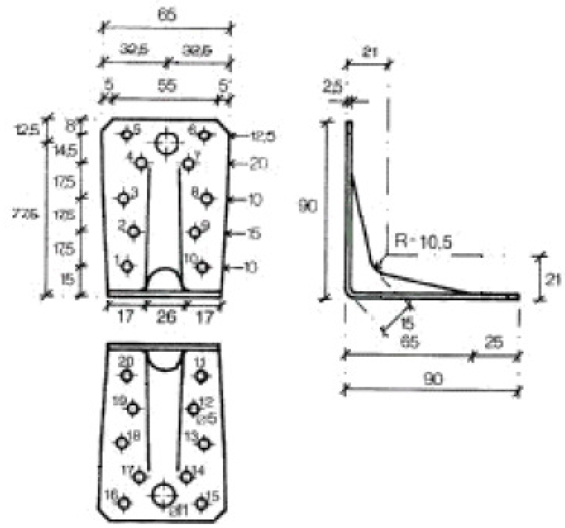


Figure 2 – Geometry of angle bracket

3.2 Angle bracket – capacities

3.2.1 Yielding and ultimate stresses

Steel quality S250GD in accordance with EN 10326.

Minimum yielding stress: 250 MPa.

Minimum ultimate stress: 330 MPa.

These stresses are used as characteristic values.

3.2.2 Bending capacity of the plane parts of the angle bracket

The characteristic yielding moment is calculated from a net cross section by reducing the width b (65 mm) by the net sum of the hole diameters (18 mm).

$$M_k = W_{pl} f_{y,k} = ¼\, b_{net}\, t^2 f_{y,k} =$$

$$¼(65\text{-}18)2{,}5^2\ 250/10^3 = 18{,}4\text{kNmm.}$$

3.2.3 Bending capacity of cross sections with the rib

The bending capacity of the cross sections with the rib have been determined by methods in accordance with the ETAG 015.

The characteristic maximum bending moments for the two positions of the applied force have been determined in accordance with the Danish Timber Code DS 413.

Position of applied force	inner	outer
e (mm)	32,5	67,5
M_k (kNmm)	287	173

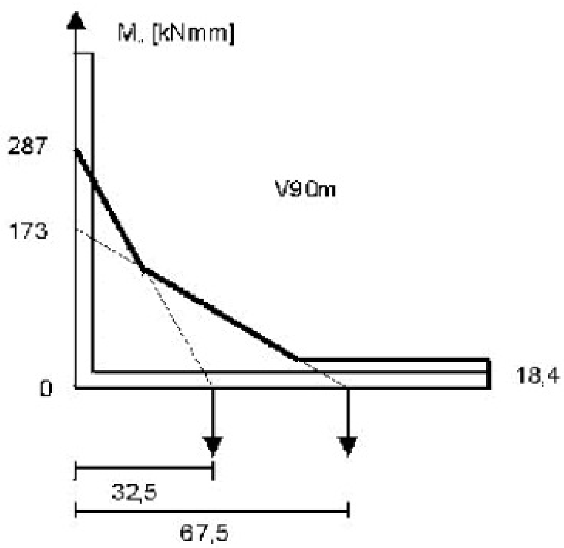


Figure 3 – Forces on angle bracket

3.3 Nail capacities

3.3.1 Lateral load-carrying capacity

According to the Danish Timber Code DS 413, the characteristic capacity is

$$F_{90,k} = 1{,}25 \cdot 135\ d^{1,7} = 1{,}25 \cdot 135 \cdot 4^{1,7} = 1{,}78\ \text{kN}$$

Since this capacity has been documented by tests with the nails used for the connection, this value will be used instead of the smaller value given in Eurocode 5 : 1994.

$$f_{h,k} = 0{,}082 \cdot ro \cdot d^{-0,3} = 17{,}3\ \text{MPa}$$

$$M_{y,k} = 180 \cdot d^{-0,3} = 6{,}62\ \text{kNmm}$$

$$F_k = 1{,}1 \cdot (2 \cdot M_{y,k} \cdot f_{h,k} \cdot d)^{0,5} = 1{,}05\ \text{kN}$$

3.3.2 Withdrawal capacity

According to a Danish MK approval supported by tests, the withdrawal capacity can be calculated from:

$$F_{ax,k} = 7{,}8 \cdot d \cdot l \text{ (see Table 1)}$$

where

7,8 = parameter (MPa)

d = diameter of the nail

$l = l_{kam}$ = the length of the threaded part.

	d (mm)	l_{kam} (mm)	l_{max} (mm)	$f_{u,k}d$ (N/mm)	$F_{ax,k}$ (kN)
40/40	4.0	30	24	31,2	0,75
40/50	4.0	40	34	31,2	1,06
40/60	4.0	50	44	31,2	1,37
40/75	4.0	65	59	31,2	1,84
40/100	4.0	70	64	31,2	2,00

Table 1

4 Lifting force on connection with two 90° angle brackets

4.1 Nailing pattern and geometry

The nailing pattern is shown in Figure 5.

Nails used: 4 mm diameter x 60 mm long.

Number of holes with nail:

- purlin: 2, 3, 4, 5, 6, 7, 8, 9

- beam: 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

4.2 Static model and calculations

The position of the forces in the connection is given in Figure 4.

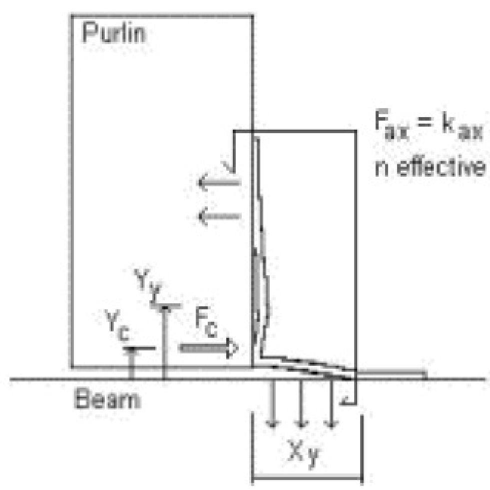


Figure 4 – Position of forces

k_{ax} = efficiency factor; $F_{ax,d}$ = characteristic withdrawal capacity of the nails in a group of n nails (where n is the number of effective nails). y_c has been assumed to be 10 mm, so wane is not allowed at the connection.

The static model described in Figure 4 has been verified from testing of the angle bracket.

The tests document that the efficiency factor k_{ax} = 0,83 is conservative.

The equations are derived for vertical equilibrium and moment equilibrium of the horizontal leg.

It is assumed that the bending moment can be carried by the two cross sections of the legs.

The moments are denoted by the symbols M_{beam} and M_{purlin}.

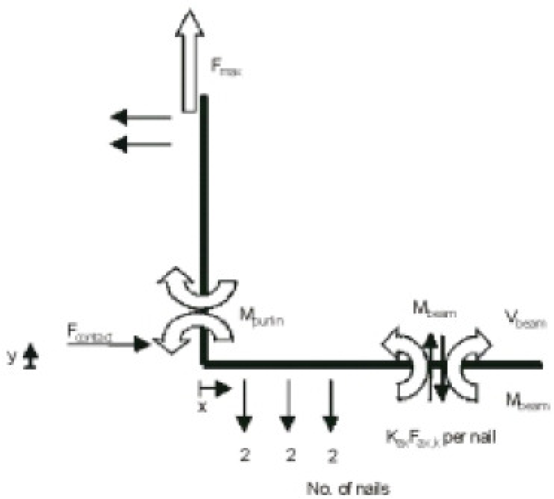


Figure 5 – Force and nail pattern

Horizontal leg:

$F_{max} + V_{beam} = n\, k_{ax}\, F_{ax,k}$ (4.1)

$M_{purlin,k} + M_{beam,k} = F_{max}x_y - k_{ax}\, F_{ax,k}\, (n\, x_y - \Sigma x)$ (4.2)

F_{max} = maximum force per angle bracket

n = number of efficient nails = 6

k_{ax} = efficiency factor for axially loaded nails = 0,83

$F_{ax,k}$ = withdrawal capacity of one nail 4,0 x 60 mm = 1,37 kN

$M_{purlin,k}$ = moment capacity for y_y = 0 mm. $M_{purlin,k}$ = 287 kNmm

$M_{beam,k}$ = moment capacity for x_y = 65 mm. $M_{beam,k}$ = 18,4 kNmm

Σx = sum of x-coordinates for nails: $\Sigma x = 2 \cdot (15 + 32{,}5 + 50) = 195$ mm

F_{max} = derived from (4.2) and from (4.1)

$F_{max} = [M_{purlin,k} + M_{beam,k} + k_{ax}\, F_{ax,k}\, (n\, x_y - \Sigma x)] / x_y = 8{,}1$ kN (4.3)

$V_{beam} = n\, k_{ax}\, F_{ax,k} - F_{max} = -1{,}3$ kN (4.4)

The value of V_{beam} is negative, indicating that the withdrawal capacity of the nails is insufficient to carry the calculated force of 8,2 kN (because a low density has been assumed for $F_{ax,k}$).

So the static model is changed for another, where it is assumed that the moment capacity in the angle of the bracket is sufficient to carry the load from the six nails in axial withdrawal.

$$F_{max} = n\, k_{ax}\, F_{ax,k} = 6 \cdot 0{,}83 \cdot 1{,}37 = 6{,}8 \text{ kN}$$

The moment in the angle of the bracket shall fulfil the equation

$M_{corner} = M_{purlin,k}\, (y_y = 0) = 287$ kNmm (4.5)

where

$M_{corner} = k_{ax}\, F_{ax,k}\, \Sigma x = 221$ kNmm (4.6)

Therefore, the requirement of the bending capacity of the angle of the bracket is fulfilled.

Vertical leg

The axial forces in the nails, Nos. 3, 4, 7 and 8, are determined by moment equilibrium for a known corner moment

$M_{corner} = F_{ax}\, \Sigma\, (y - y_c) = F_{ax}\, (\Sigma y - n y_c)$ (4.7)

resulting in

$F_{ax} = M_{corner} / (\Sigma y - n y_c) = M_{corner} / (235 - 4 \cdot 10) =$

$M_{corner} / 195 = 1{,}13$ kN $= F_{ax,k} = 1{,}37$ kN (4.8)

The lateral forces in the other nails, Nos 2, 5, 6 and 9, are given by

$F_{90} = F_{max}/n = 1{,}7$ kN $= F_{tv,d} = 1{,}78$ kN(n = 4) (4.9)

Conclusion

The characteristic load-carrying capacity for a connection with two 90° angle brackets with 4 mm diameter x 60 mm long nails in the holes indicated above and without wane, subjected to a lifting force, is

$$F_{max,k} = 2 \cdot 6{,}8 = 13{,}6 \text{ kN}$$

5 Shear force in the direction of the purlin

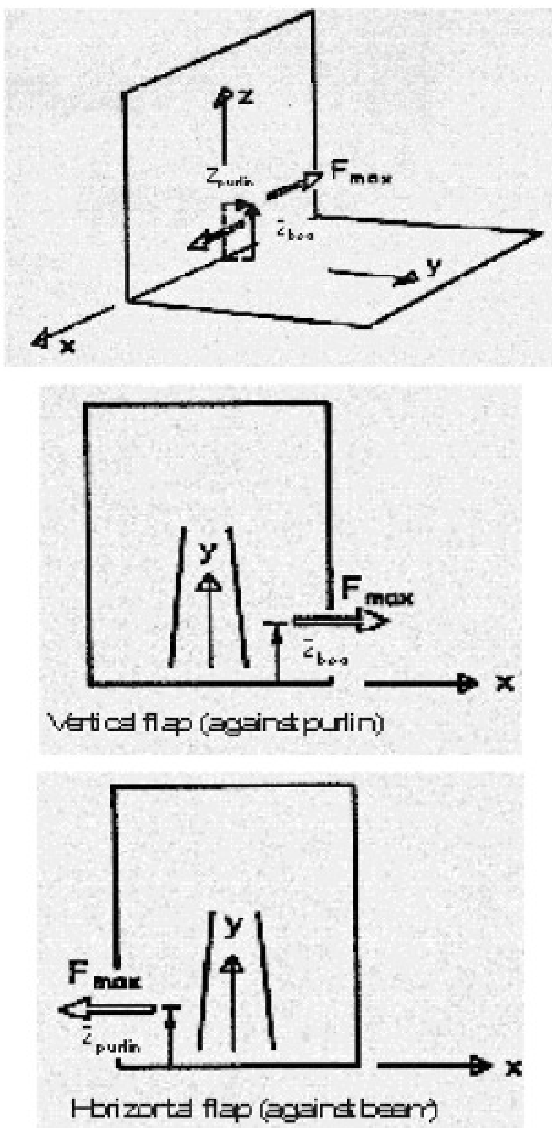
5.1 Basic equations

The force is transferred from the purlin to the vertical leg by lateral force in the nails.

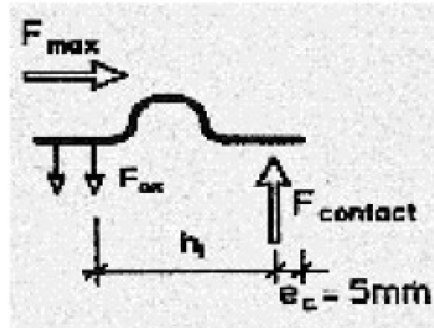
Therefore, the nail group is loaded eccentrically.

Some of the nails close to the angle will also be loaded by axial forces, since the resulting force on the vertical leg is situated a distance z_{purlin} over the horizontal leg.

Similarly the force is transferred through the horizontal leg and the nails holding it.



Forces in the plane of the flaps



Tensile forces in the nails from the torsional moment

Figure 6 – The principle transfer of forces in an angle bracket connection

For each nail group and leg, the lateral and axial forces in the nails can be determined.

Computer analyses of an elastic, plane nail group show that for a certain range of eccentricity (z_{purlin} or z_{beam}), a relationship between the force F_{max} and the maximum lateral force on a nail (F_{90}) can be determined.

This is true for both the vertical and the horizontal leg:

$$F_{90} = (k_0 - k_1 Z_{purlins})\, F_{max} \qquad (5.1)$$

Using two calculations of the same plane elastic nail group the parameters k_0 and k_1 can be determined.

The axial force in the nails of the vertical and the horizontal legs respectively at the angle is calculated from the equations

$$F_{ax,\, ver} = F_{max}\, Z_{purlin} / \Sigma h_{l,ver} \qquad (5.2)$$

$$F_{ax,\, hor} = F_{max}\, Z_{beam} / S_{hi,\, hor} \qquad (5.3)$$

By combining equations (5.1) to (5.3), the following two inequalities can be formulated for the vertical and the horizontal legs.

From the failure criterion given in Eurocode 5 for combined axial and lateral load on a nail, the inequalities are

Vertical leg:

$$F_{max} \sqrt{\left[\frac{k_{0,\,ver} - k_{1,\,ver}\, z_{beam}}{F_{90,k}}\right]^2 + \left[\frac{z_{purlin}}{\Sigma h_{i,\,ver} F_{ax,\,k}}\right]^2} \leqslant 1.0$$

Horizontal leg:

$$F_{max} \sqrt{\left[\frac{k_{0,\,hor} - k_{1,\,hor}\, z_{purlin}}{F_{90,k}}\right]^2 + \left[\frac{z_{beam}}{\Sigma hi,\, hor Fax,\, k}\right]^2} \leqslant 1.0$$

When the left sides of the inequalities are equal to 1,0, failure occurs in both legs.

Should failure occur in one leg, a catastrophic failure may not occur but the other leg will be loaded more heavily.

The axial force in the nails causes bending moments in the leg.

Moment capacity has been determined conservatively, it is a minimum value.

$$M_{perp,min} = 76 \text{ kNmm}$$

This moment capacity shall be checked to ensure it is not exceeded.

5.2 A connection with a single 90° angle bracket with a rib

Since the purlin is connected to the beams below at both ends, the force will be transferred from the purlin in the position shown in the previous example.

Extra eccentricity moments do not occur.

A full nailing pattern is employed, as shown in the example given in clause 4.1 and Figure 2.

The parameters of equation (5.1) have been determined by a linear elastic analysis with arbitrarily chosen eccentricities of 10 mm and 30 mm repectively.

For an external force of 1,0 kN, the maximum lateral force in a nail becomes

<u>Vertical leg:</u>

e = 10 mm   30 mm $k_{0,ver} = 0{,}434$

F_{90} = 0,3796kN   0,2701 kN $k_{1,ver} = 0{,}00548 \text{ mm}^{-1}$

<u>Horizontal leg:</u>

e = 10 mm   30 mm $k_{0,hor} = 0{,}303$

F_{90} = 0,2613 kN   0,1774 kN $k_{1,hor} = 0{,}0042 \text{ mm}^{-1}$

The axial forces in the nails are determined from equations (5.2) and (5.3), with

<u>Vertical leg:</u>

$\Sigma h_{i,ver} = 15 + (30 - 5) = 40$ 1 effective nail

<u>Horizontal leg:</u>

$\Sigma h_{i,hor} = 20 + 15 + 2 \cdot (30 - 5) = 85$ 2 effective nails

In the tests, the z-values were found to be:

$$z_{beam} = 10 \text{ mm}$$

$$z_{purlin} = 18 \text{ mm}$$

In the tests, the nailing pattern was equal in both legs and the difference has its origin in the different capacities of the nails in the beam and in the purlin.

In this situation there are fewer nails in the vertical leg, so z_{purlin} shall be reduced as the torsional stiffness of this leg is less.

The following result agrees with this.

The Table 2 gives the values of F_{max} for some selected values of eccentricities. F_{max} has been determined from equations (5.4) and (5.5) by making the balancing of the inequalities and solving for F_{max}:

Nails: $F_{90,k} = 1{,}78$ kN
$F_{ax,k} = 1{,}37$ kN

z_{beam} (mm)	z_{purlin} (mm)	$F_{max,ver}$ (kN)	$F_{max,hor}$ (kN)	$F_{ax,ver}$ (kN)	$F_{ax,hor}$ (kN)	M_{ver} (kNmm)	M_{hor} (kNmm)
10	18	2.55	6.50	1.149	0.764	35.92	34.39
14	6	4.37	5.08	0.656	0.836	20.50	37.62
16	4	4.81	4.73	0.481	0.890	15.04	40.05
15	4	4.75	4.85	0.475	0.857	14.83	38.55
15.5	4	4.78	4.79	0.478	0.874	14.93	39.31

Table 2 – Iteration table

The characteristic load-carrying capacity for a shear force is 4,7 kN.

The moment capacity for M_{perp} in the legs is checked for F_{max} = 4,7 kN

$F_{ax,ver} = F_{max}\, z_{purlin} / \Sigma h_{i,ver} = 3{,}30 \cdot 2 / 42{,}5 = 0{,}47$ kN

$M_{ver} = \Sigma(F_{ax,ver}(y_{nail,ver} - t/2)) = 0{,}47 \cdot (32{,}5 - 1/2 \cdot 2{,}5) = 14{,}7 \text{ kNmm} < M_{perp,k} = 76{,}0 \text{ kNmm}$

$F_{ax,hor} = F_{max}\, z_{beam} / \Sigma h_{i,hor} = 3{,}30 \cdot 14{,}1 / 87{,}5 = 0{,}86$ kN

$M_{hor} = \Sigma\, [F_{ax,hor}(y_{nail,hor} - t/2)] = 0.86 \cdot (15 + 32{,}5 - 2 \cdot 1/2 \cdot 2{,}5) = 40{,}7 \text{ kNmm} < 76{,}0 \text{ kNmm}$

It is shown, therefore, that each leg has sufficient bending capacity.

<u>Conclusion</u>

It can be concluded that the characteristic load-carrying capacity for a connection with one 90° angle bracket with 4 mm diameter x 60 mm long nails in the holes indicated above and without wane, subjected to a shear force in the direction of the purlin, is:

$$F_{max} = 1 \cdot 4{,}7 = 4{,}7 \text{ kN}.$$