EOTA

TECHNICAL REPORT

Principle for the static calculation of connections made with three dimensional nailing plates, with examples

TR 015

Edition February 2002

EUROPEAN ORGANISATION FOR TECHNICAL APPROVALS

Principles for the static calculation of connections made with three dimensional nailing plates, with examples

Edition February 2002

Foreword

EOTA Technical Reports are developed as supporting reference documents to European Technical Approval Guidelines and can also be applicable to a Common Understanding of Assessment Procedures, an EOTA Comprehension Document or an European Technical Approval, as far as reference is made therein.

EOTA Technical Reports go into detail in some aspects and express the common understanding of existing knowledge and experience of the EOTA bodies at a particular point in time.

Where knowledge and experience is developing, especially through approval work, such reports can be amended and supplemented.

When this happens, the effect of the changes upon the European Technical Approval Guidelines will be laid down in the relevant comprehension documents, unless the European Technical Approval Guideline is revised.

This EOTA Technical Report has been prepared by the EOTA Working Group 06.03/01 – "Three dimensional nailing plates" and endorsed by EOTA.

1 Scope

This Technical Report gives principles for, and examples of, the static calculation of connections made with three-dimensional nailing plates, with examples, in timber structures.

2 Introduction

The technical literature from manufacturers of three-dimensional nailing plates normally includes tables which show the product's load-carrying capacity for a particular nailing configuration. In calculations it is necessary to ensure that the proposed nailing configuration is in accordance with the manufacturer's assumptions, and that the static models assumed are in accordance with the proposed use in the timber structures.

3 Design principles

The basic principles for the calculation of connections with three-dimensional nailing plates are illustrated with some typical examples:

3.1 Beam to beam connection

Since there will normally be a gap between the two beams in the connection, the reaction force needs to be transferred by the angle brackets only, see Figure 1.

This method is applicable to thin steel plates since these result in low stiffness and yielding strength of the flanges for both torsion and plate bending.

In this way it is ensured, that each flange of the nailing plate will be subjected to a shear force very close to the corner.

If the force is positioned elsewhere, the torsional yielding strength would be exceeded, the nailing plate would yield resulting in the force ending in the corner.

The two groups of nails can be calculated independently of each other as eccentric loaded nailing plates where the ductility of the laterally loaded nails is utilised.

If the nailing plates are thick and hence have a large torsional stiffness and load carrying capacity, there will be a risk of withdrawal of the nails or screws causing a progressive failure (a zipper like failure).

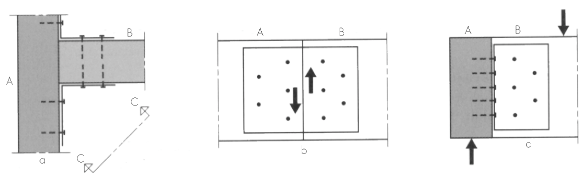

Figure 1 - Beam to beam connection. Connection with two angle brackets and nails or screws.
a = plan; b = view on c-c (expanded); c = elevation; A = Cross beam; B = Beam

This simple static model may be refined by allowing for the occurrence of torsion in the thin steel plates and withdrawal forces in the fasteners.

In this case the load-carrying capacity of the fasteners shall be verified for a combination of actual and lateral force.

3.2 Purlin with purlin anchors

In this example, shown in Figure 2, an analysis of a purlin connected to a beam with purlin anchors is carried out.

The illustration shows how the design of the structure influences the actions on the nailing plates.

It is assumed that the resulting force from the loading case “wind suction on the roof” acts in the middle of the purlin.

The ability of the roof cladding to move the resulting wind force due to extra tensile force in the nail or screw and contact pressure between the purlin and the cladding is disregarded, this assumption is reasonable for thin corrugated roof sheets with screws in the middle of the purlin.

As in the former example it is assumed that the force transfer in the nailing plate corresponds to a shear force in the corner of the nailing plate.

If the purlin anchors are placed on the same side of the purlin (Figure 2 upper drawing) equilibrium requires that the nail group in the beam is able to carry the eccentricity moment from the force in the middle of the purlin.

If the purlin anchors are placed diagonally in relation to the purlin (Figure 2 lower drawing) the eccentricity moment will be much smaller, however, if the purlin is subjected to a torsional moment, it does not normally present any problems.

If two purlin anchors are used per connection, as shown on the dotted nailing plate in the section in Figure 2 it is possible to obtain equilibrium by the inclined forces in the nails in the beam.

In this way the nail groups are subjected to a central force without any large withdrawal forces in the nails.

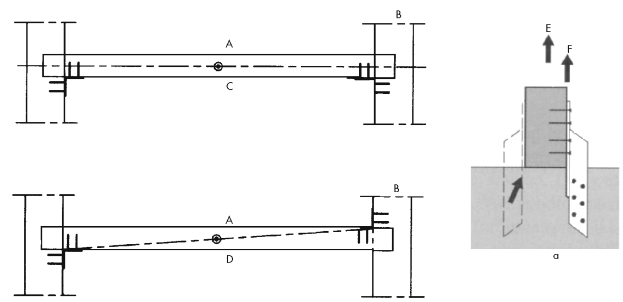

Figure 2 - Anchoring of a purlin by means of purlin anchors and nails
a = section; A = Purlin; B = Beam; C = Purlin anchors placed on the same side; D = Purlin anchors placed diagonally; E = Suction, purlin anchors placed on the same side; F = Suction, purlin anchors placed diagonally

3.3 Purlin with angle brackets

In this example a purlin is analysed for a lifting force and connected to an underlying beam with angle brackets.

The photo in Figure 3 illustrates the static behaviour of the angle brackets and the nails which are subjected to withdrawal forces.

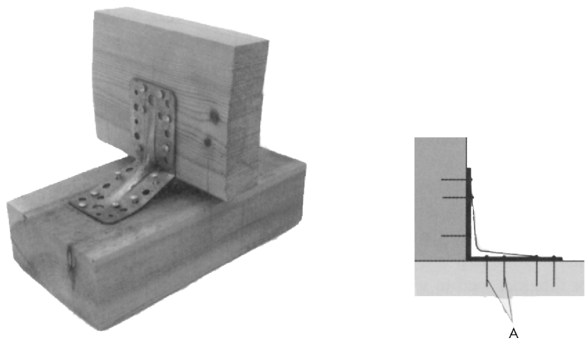

Figure 3 - Failure in a connection with an angle bracket and annular ringed nails subjected to a lifting force.
Only the inner nails, which are pulled out successively, are active.
A = Force transfer

Only the inner nails are active; the outer nails are not subjected to a withdrawal force, as a yielding hinge is formed in the thin flange of the bracket.

(Stiffer and stronger angle brackets exist where nails are simultaneously active for withdrawal.)

A single bracket subjected to a lifting force as shown in Figure 4 transfers the forces by axial tension in the nails near the vertical flange and by contact compression near the free edge of the horizontal flange.

The necessary expressions for the determination of the withdrawal force are given in Figure 4.

The eccentricity *e* is given as the distance from the action line of the force, and a_c is a small length.

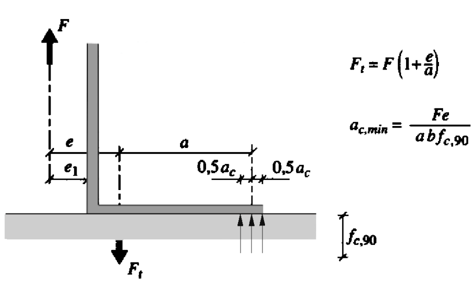

Figure 4 - Angle bracket subjected to a lifting force.
The withdrawal force can easily be found from the expressions when the small length a_c is estimated. Symbols are defined in Eurocode 5.
B = Width of bracket

If one angle bracket is considered, withdrawal forces in the nails in the horizontal flange are calculated using e_1, which corresponds to half the width of the purlin (unless it can be demonstrated that the roof covering affects the line of action of force F).

In the vertical flange the nails shall be able to transfer the moment F_{e1} together with the lateral load *F.*

Where the nails are placed in a row, an assessment has to be made of the extent the nails act together in transferring axial forces.

In this case the distances between the nails, the stiffness of the bracket as well as the axial stiffness of the nails and possibly the forming of a yielding hinge have to be considered.

For the connection shown in Figure 3 tests show that it is not possible to take account of more than two closely spaced rows of annular ringed nails acting together to transfer the axial withdrawal force.

In Figure 5 two connections with angle brackets are shown, where it is possible to find an estimate of the active forces, which is close to reality.

In both cases the vertical force is distributed evenly over the nails in the vertical flange.

If two angle brackets are considered (or diagonally positioned, see clause 3.2) the calculation of withdrawal forces in the nails in the horizontal flange may be undertaken with e = o.

The shifting of position of the force F gives a moment in the vertical flange as indicated in the diagrams over the forces and the internal moments - but in practice this can be disregarded.

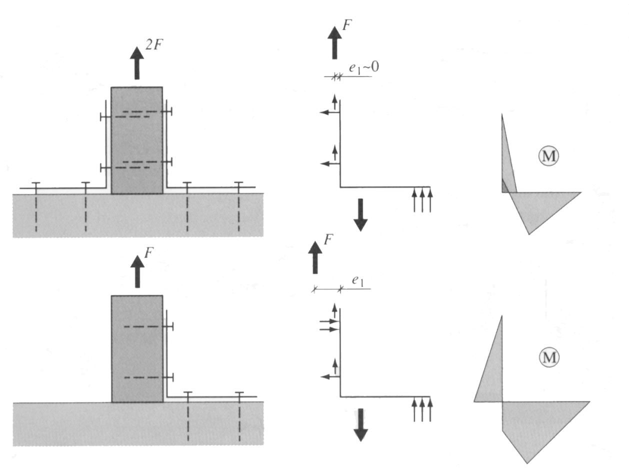

Figure 5 - Connection with double and single angle brackets.
Internal forces and moments in the bracket.

3.4 Transfer of forces by contact compression stresses

When a force is transferred by contact stresses between the nailing plate and the timber member, bending will always occur in the nailing plate.

Plate bending can best be modelled by the theory of plasticity, which is covered in clause 4.

A good (and safe) estimation of the load carrying capacity can be obtained by assuming that the contact stress is equal to the compression strength of the wood perpendicular to the grain $f_{c,90}$ and that it is placed within the given geometry resulting in the largest load-carrying capacity.

The compression strength $f_{c,90}$ for the relatively small contact area can be increased as described in the revision to EC5.

For the joist hanger with sharp edge timber in Figure 6 the loaded width *a* can be found from moment equilibrium, since the yielding plate moment of the nailing plate is known to be m_y (per length unit).

A

B

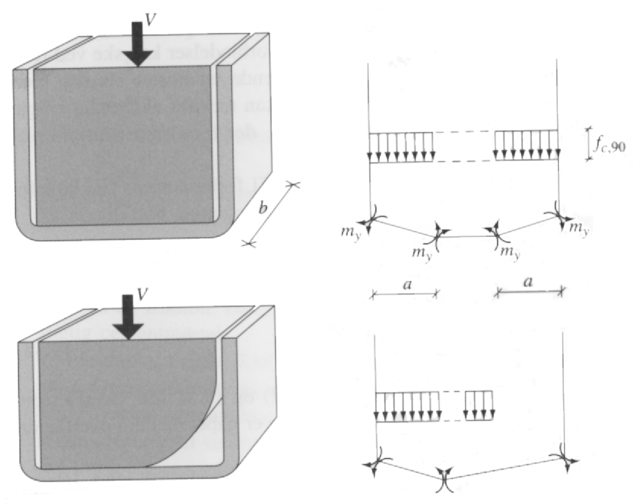

Figure 6 - Contact compression stress at the bottom of a joist hanger.
A = Sharp edge timber; B = Timber with wane

$$2m_y = \frac{f_{c,90}a^2}{2}$$

$$a = 2\sqrt{\frac{m_y}{f_{c,90}}}$$

where equilibrium in the direction of the shear force results in

$$V = 2ab \cdot f_{c,90} = 4b\sqrt{m_y \cdot f_{c,90}}$$

It is assumed that the yielding moment is the same at the bottom and the sides of the joist hanger, and that they have the same width *b*.

Where this is not the case, e.g. where different plate thicknesses are used in a welded joist hanger, it is simple to take this into account.

If the timber does not have sharp edges at the support, this will result in another (less favourable) load distribution, see the lower drawing in Figure 6.

4 Plastic design principles

This section deals with plane connections with fasteners for which the force-deformation relation can be assumed to be perfect plastic ie with a horizontal force deformation relationship.

Only statically determinate connections are treated, but extending the philosophy to statically indeterminate connections is not difficult.

For simplicity only two dimensional nailing plates are considered although these are outside the scope of the ETAG but three dimensional solutions may frequently be modelled by two dimensional considerations.

It is assumed that the force-deformation relation of the fasteners is rigid-plastic, but the expressions are also applicable to elastic perfectly plastic fasteners, when it can be demonstrated, that all fasteners considered are on the perfectly plastic (ie the horizontal) part of the force-deformation relation.

For a connection with plastic fasteners one can calculate an upper bound and a lower bound for the exact plastic load carrying capacity.

Frequently it is harder to determine the latter, and since it is relatively easy to find an upper bound, which is close to the exact plastic capacity, one can frequently be satisfied with an upper bound.

4.1 Determination of an upper bound

An upper bound of the load carrying capacity of a connection can be determined in the following way:

- a rigid body deformation is assumed;
- it is assumed that the force in a fastener is equal to the yield force and that the direction of the force is equal to the direction of the relative deformation;
- an upper bound of the plastic load carrying capacity is calculated by putting the internal work W_{inner} of the fasteners equal to the external work W_{ex} of the external force. (The principle of Virtual Work).

Frequently it is convenient to describe the rigid body motion as a rotation about an estimated rotation centre.

As a guide for the estimation the exact centre of rotation is often situated close to a line perpendicular to the external force R through the elastic centre of gravity of the connection, see Figure 7.

The centre of rotation often lies opposite the force R in relation to the centre of gravity; the bigger the eccentricity the closer to the centre of gravity.

The virtual rotation is denoted *θ*. With the symbols in Figure 7 the internal work and the external work can be found from:

$$W_{inner} = \sum_{i=1}^{n} \theta r_i F_y$$

$$W_{ex} = \theta e R$$

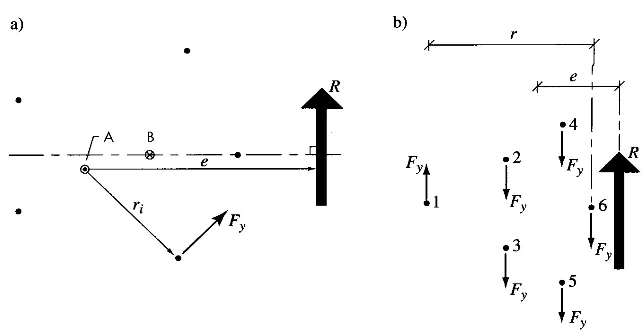

Figure 7 - Determination of upper bound (a) and lower bound (b).
The yield force of a fastener is denoted F_y.
A = estimated centre of rotation; B = Centre of gravity.

The upper bound denoted as R^+ can be calculated from

$$R^+ = \frac{\sum_{i=1}^{n} \theta r_i F_y}{\theta e} = \frac{\sum_{i=1}^{n} r_i F_y}{e} = \frac{F_y}{e} \sum_{i=1}^{n} r_i \qquad (1)$$

Equation (1) can be perceived as a moment equilibrium equation about the estimated centre of rotation.

The following examples show, that R^+ will only be slightly larger than the exact value provided a reasonable centre of rotation is employed.

It is assumed for the plastic calculation, that all fasteners contribute together with their full yielding force.

This requires that the slip at the fasteners close to the centre of rotation is so large, that the yielding force has been reached.

The fasteners far away from the centre of rotation can be subjected to large slips, which finally can be so large that failure will occur.

Where splitting will not occur, based from experience in tests, one can normally assume that

$$u_{failure} = 4u_y$$

where:

$u_{failure}$ = the slip (relative displacement) at failure;

u_y = the slip at the beginning of yielding.

Therefore, it will normally be a conservative estimate to disregard the fasteners, which have

$$r \leq 0{,}25\, r_{max}$$

where r = distance to the centre of rotation;

r_{max} = the maximum distance for a fastener from the centre of rotation (see Figure 7).

The reduction of the load carrying capacity of the connection will normally be insignificant, and in practical calculation this is often disregarded.

4.2 Determination of a lower bound

A lower bound of the load carrying capacity of a group of plastic fasteners can be determined by estimating a force distribution over the fasteners in a way that the equations of equilibrium are fulfilled (i.e. a static allowable force distribution), and that the force in every single fastener is less than or equal to the yield force (i.e. a secure force distribution).

4.2.1 Example of the determination of a lower bound

As an example the lower bound of the connection shown in Figure 7b is determined.

It is assumed that some of the fasteners give equilibrium in the direction of the force, (fasteners 2-5) and that the others give moment equilibrium, (fasteners 1 and 6).

The easiest way to set up the moment equilibrium is by taking moment about the resultant of the fasteners providing force equilibrium, (fasteners 2 - 5).

The distance between this resultant and *R* is denoted as *e* in the figure.

A lower bound, R^- , of the load carrying capacity of the connection is given by:

$$R^- = \min\left\{4F_y, \frac{r}{e}F_y\right\}$$

It should be noted that with the estimated force distribution shown in Figure 7b equilibrium perpendicular to the outer force is fulfilled automatically, since all forces are assumed parallel to this.

4.3 Plastic calculation examples

4.3.1 Example 1

This example concerns the plastic load carrying capacity of a fish plate connection in a truss cord shown in Figure 8 where the 14 nails connect the end of the fish plate to the cord.

It is assumed that $N = 2V$.

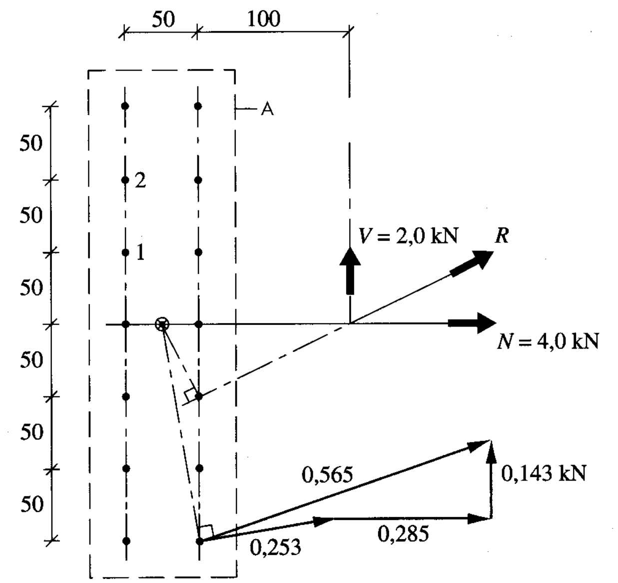

Figure 8 - Nail connection subjected to an eccentric force.
All dimensions in mm.
A = Boundary of fictitious continuous layer; B = Centre of gravity

The elastic load carrying capacity from proportional calculation has been determined as

$$V = 0{,}58/0{,}565 \cdot 2{,}0 = 2{,}05 \text{ kN}$$

where:

0,58 = design load-carrying capacity of the nail;

0,565 = calculated design force (see Figure 8).

Equation (1) is applied to calculate an upper bound of the load carrying capacity of the perfectly plastic connection. It is assumed that the centre of rotation is situated in nail number 1.

$$\Sigma r_i = 2\,(50 + 100) + 150 + 200 + 50 + 2\,(71 + 112) + 158 + 206 = 1430 \text{ mm}$$

By measuring or calculation, $e = 112$ mm, and equation (1) results in

$$R^+ = \frac{0{,}58 \cdot 1430}{112} = 7{,}41\text{kN}$$

which is equivalent to

$$V^+ = 7{,}41/\sqrt{5} = 3{,}31\text{kN}$$

If the centre of rotation is assumed to be at nail number 2, one finds $V^+ = 2{,}86$ kN, which is close to the exact load carrying capacity of 2,81 kN, determined from an iterative calculation.

A lower bound of the load carrying capacity of the perfectly plastic connection can be determined by assuming, that the forces in the middle 10 nails are parallel to *R*, and that the forces in the outer 4 nails are perpendicular to the radius from the centre of gravity.

With this force distribution equilibrium perpendicular to *R* is fulfilled; it is statically allowable.

Requiring that the forces in the nails shall be less than or equal to the yielding force one gets:

$$R^{-} = V^{-} \cdot \sqrt{5} \leq 2 \cdot 5 \cdot 0{,}58 \qquad V^{-} \leq 2{,}59 \text{ kN}$$

$$M^{-} = V^{-} \cdot 125 \leq 4 \cdot 153 \cdot 0{,}58 \qquad V^{-} \leq 2{,}84 \text{ kN}$$

So, the load carrying capacity of the perfectly plastic connection is limited by

$$\sqrt{5} \cdot 2{,}59 \leq R_y \leq \sqrt{5} \cdot 2{,}86 \text{ kN}$$

4.3.2 Example 2

Figure 9 shows a fish-plate subjected to a shear force with a direction as shown.

A force distribution is assumed, where the force in all nails is parallel to the line connecting the centres of gravity and with a magnitude equal to the yield force.

Equilibrium perpendicular to *V* is ensured by the contact force F_c between the ends of the beams.

Friction is disregarded.

$$V^{-} = 2 \cdot 10 \cdot 0{,}73 \cdot \cos\alpha = 11{,}7 \text{ kN}$$

Since the estimated force distribution can be achieved by a set of translations and rotations of the members connected, forming a geometric possible deformation field, this force is also an upper bound, i.e. it is an exact plastic solution.

If the shear force changes direction, the system will change, because equilibrium can no longer be achieved by the contact pressure.

The nail groups will be subjected to an eccentric load *V*.

The plastic load carrying capacity V_y has been calculated as 10,6 kN.

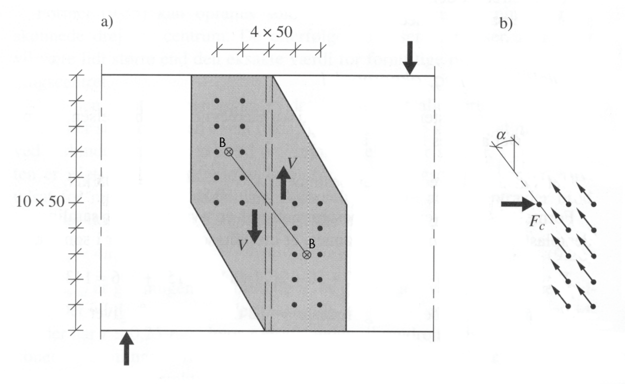

Figure 9 - Connection with a fish-plate on each side.
Ten nails 31/80 per group with a design load carrying capacity of 1,25 · 0,58 = 0,73 kN per nail, per shear plane (Factor 1,25 for steel to wood).
B = Centre of gravity

4.3.3 Example 3

This example deals with a gusset plate connection. It is assumed that there is a good fit between the timber members, i.e. the joint is small with nails as fasteners distributed evenly and with the same number *n* in each group.

The rotation will press the top chord and the bottom chord together, so a contact force will emerge in the joint.

Figure 10 shows the forces and deformations after yielding in the nails has developed.

Friction in the joint is disregarded, and it is approximated that each nail group is loaded centrally with a force *F*.

Further, it is assumed that the component of *R* perpendicular to the joint results in a contact force.

By projection it can be found, that *F* can be expressed by R_{par}, which is the component of *R* parallel to the joint.

$$F = R_{par} / \cos(\alpha + \beta)$$

where the angle *β* has been determined from frame analyses and it can approximately be determined from

$$\tan\beta = 0{,}2/(0{,}7 \cdot l/h - 0{,}3 \cdot \cot\alpha)$$

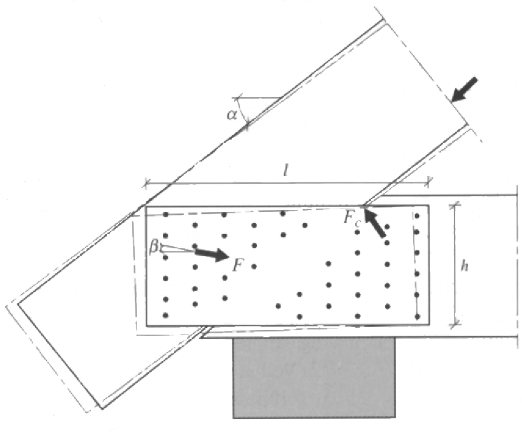

Figure 10 - Gusset plate connection between top chord and bottom chord.

There is a good fit, so a contact force emerges in the deformed shape shown with dotted lines, the forces acting on the top chord are shown with arrows. The thickness of the gusset plate is *b*.

The strength of the gusset plates should be verified for a tensile force F_t or a shear force F_v.

$$F_t = \frac{R_{par}}{\cos\alpha} \leq A_{gusset} f_t \qquad \text{for} \qquad \tan\alpha \leq \frac{f_v}{f_t}$$

$$F_v = R_{par} \leq \frac{A_{gusset}}{\sin\alpha} f_v \qquad \text{for} \qquad \tan\alpha > \frac{f_v}{f_t}$$

where:

A_{gusset} = cross sectional area *b h*.

By verification of the shear strength a failure plane parallel to the joint should be used.

This description of calculation methods for plastic plane connections can be utilised for the static analysis of three dimensional nail plate connections where the internal forces are transferred in the plane of the thin plates of the brackets.

The beam to beam connection shown in Figure 1 is a typical example.